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FAST TRACK COMMUNICATION

Shape instability of a magnetic elastomer membrane

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Abstract
Shape instability occurring in a thin plate (membrane) made of a soft magnetoelastic material under a uniform magnetic field is predicted and analysed. The instability onset is shown to be similar to the second-order transition; the dependence of the threshold field on the magnetic and geometric parameters of the membrane is derived analytically; the membrane shapes (domes) are evaluated with the aid of numerical simulation. The theory proposed is in general agreement with experiments performed on a siloxane rubber/iron carbonyl composite.

Soft magnetic elastomers (SMEs) are weakly linked high-elasticity matrices filled with magnetic particles of micrometre or submicrometre size. A remarkable functional property of these materials is the giant magnetic strain effect reaching tens of per cent [1–6]. Due to this, SMEs are envisaged for many applications [4, 5, 7, 8]. Magnetic stresses exerted on the dispersed particles and transferred via them to the matrix, (i) in a uniform field tend to stretch the body in the direction of the field and (ii) in a non-uniform field crave to move and stretch it in the direction of the maximal field intensity. This tendency is opposed by elasticity that strives to preserve the initial shape of the body. However, due to SME softness, a magnetized sample achieves the state of the minimal free energy via substantial configuration changes, i.e. giant magnetostriction.

The magnetic interactions, which induce strain of SMEs, have a dipole–dipole origin and thus are long ranged. As a result, the observed manifestations of the magnetostriction effect depend importantly on the sample shape and on the orientation of the sample with respect to the magnetizing field. In papers [9–12] we have analysed the magnetic striction of SME spheres, spheroids and spherical capsules. The subject of this work is the behaviour of a thin flat plate (membrane) fixed along its rim and magnetized by a uniform magnetic field $\boldsymbol{H}_0$ normal to its surface.

When considering possible deformations of the membrane, we will ignore the azimuthal perturbations thus assuming that the rotational symmetry with respect to the geometric axis always holds. Meanwhile, we note that the condition of axial rotary invariance is insensitive to the existence or not of a plane of symmetry normal to the axis. In other words, under the symmetry restriction imposed the membrane may either remain flat or form some concentric pattern(s). It is reasonable to surmise that under the field a transition from a plane to a dome-like shape takes place. Indeed, in the initial state the membrane surface is perpendicular to the field that is quite unfavourable from the magnetostatic viewpoint. Provided the 'energy fee' for producing extra surface area is low enough, the membrane might strive to bulge or to develop a set of radial folds. In such patterns the sample surface is inclined to the field under more favourable angles than $\pi/2$, thus yielding diminution of the magnetic energy. A similar behaviour, which obeys the soft instability scenario, is well known for thin layers of magnetic fluids [13].

In a membrane that is fixed over its rim, the spectrum of out-of-plane deformations is discrete, and the mode of the lowest energy corresponds to a simple bulging: the formation of a dome. Assume that the membrane is positioned vertically and a uniform field $\boldsymbol{H}_0$ is horizontal; according to our experimental evidence this practically excludes the gravity effect. For this scheme the above-given considerations predict that the rightward or leftward domes would be equally probable, a situation that closely resembles the occurrence of clockwise/counter-clockwise turns of the liquid crystal director in the magnetic Friedericksz transition [14]. Degeneration of such a kind entails with necessity that the transition takes place at some finite threshold, i.e. the membrane changes its shape only when the field attains some finite value H_{0c}.

To obtain a quantitative estimate for this critical field, we set a cylindrical coordinate framework in the middle plane of the membrane in its flat state and denote by ζ the perturbations normal to this plane. The deflection energy of a thin incompressible round plate of diameter D and thickness h may be taken directly from elasticity theory [15]:

$$\delta U_{\text{elast}} \sim Eh^3 \int_0^{D/2} \left[\zeta''(\rho)\right]^2 \rho \, d\rho; \tag{1}$$

here primes denote the derivatives with respect to the radial variable ρ. On the other hand, the magnetostatic energy of the plate reduces locally with the increase in the angle α between $\boldsymbol{H}_0$ and the normal to the membrane, so that $\delta U_{\text{magn}} \propto -\alpha^2$. Taking into account that for small deflections $\alpha = \zeta'$, the magnetic energy increment is written as

$$\delta U_{\text{magn}} \sim -\mu_0 h \, [M(H)]^2 \int_0^{D/2} (\zeta')^2 \rho \, d\rho, \tag{2}$$

where H is the magnetic field inside the membrane, $M(H)$ the magnetization of a magnetically soft SME and μ_0 the magnetic constant. Thence, the total energy increment for a weakly deformed membrane is

$$\delta U \sim h \int_0^{D/2} \left(Eh^2 \zeta''^2 - \mu_0 M^2 \zeta'^2\right) \rho \, d\rho. \tag{3}$$

We introduce surface perturbations of the form of a cylindrical wave $\zeta = \zeta_0 \cos(\pi n \rho / D)$, where n is an odd integer. Substituting this in (3) and integrating over the radius, one gets

$$\delta U \sim \left[E(\pi n h/D)^2 - \mu_0 M^2\right] (\pi n \zeta_0)^2 \, h. \tag{4}$$

When the expression in square brackets turns negative, a flat membrane becomes unstable. As seen, it occurs at a finite field strength. The minimal value H_c of the critical internal field corresponds to $n = 1$; this means that at the threshold the magnetization attains the value $M(H_c) = (\pi h/D)\sqrt{E/\mu_0}$. Assuming that the material is linearly magnetizable ($M = \chi H$) and setting the demagnetizing coefficient in the plane of the membrane equal to zero so that $H = H_0/(1+\chi)$, one gets for the applied (external) field strength the estimate

$$H_{0c} \simeq \frac{\pi(1+\chi)\, h\sqrt{E}}{\chi D \sqrt{\mu_0}}. \tag{5}$$

Note that the dependence rendered by (5) complies with intuition: H_{0c} goes down with the increase in the membrane diameter and magnetic susceptibility while it grows with the membrane thickness and the elastic modulus.

For a consistent evaluation of the magnetic strains in a SME membrane both below and above the threshold, we employ a non-linear continuum model developed in [10]. The basic equations and boundary conditions are written with regard to the actual problem and then a finite-element solution procedure allowing for finite strains is used; the details of this treatment are cumbersome but secondary. The results obtained are exemplified in figure 1, where the curves describe the dimensionless displacement $|\Delta L|/h$ of the centre of the membrane from the initial plane. The material parameters correspond to the samples with the aspect ratio $D/h = 20$; the values of the magnetic susceptibility are chosen to be close to that of a real material. As seen, the onset of bulging occurs at a finite field strength, which confirms the prediction of the threshold character of the effect. In figure 2 the mesh patterns taken from simulations show the axial cross-section of the same membrane with the susceptibility $\chi = 3$ in a field that is about three times higher than the corresponding critical value. As expected, the occurring configuration has a dome-like shape. Test calculations show that for $D/h > 5$ the critical field values rendered by (5) agree very well with the numerical results on H_{0c}.

For the experiment, a set of thin SME plates was prepared with thicknesses 1–2 mm and diameters 20–50 mm. The material is obtained by admixing iron carbonyl powder (2–5 μm) in a silicone oligomer that is polymerized and plastisized; see [3, 16, 17] for details. The bulk magnetization curve of the obtained SME is quasi-linear up to 150 kA m^{-1}; the susceptibility in the range 0–100 kA m^{-1} is virtually constant and equals $\chi = 2.14$. To prepare a sample, a round

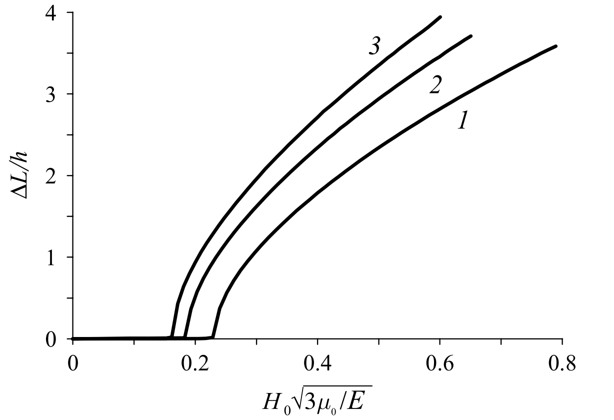

Figure 1. Dependence of the absolute value of the membrane central point displacement on the applied field strength; $D/h = 20$, magnetic susceptibility $\chi = 2$ (*1*), 3 (*2*), 4 (*3*).

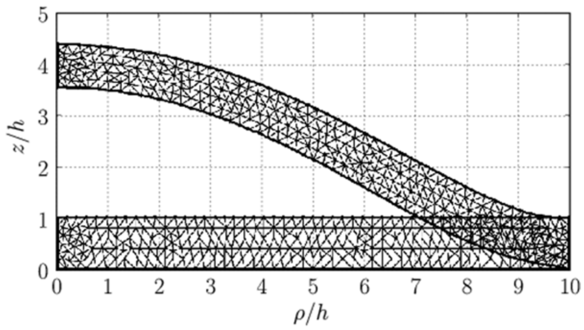

Figure 2. Profile of the axis cross-section of a membrane with $\chi = 3$ and $D/h = 20$ in the absence of the field (flat) and for $H_0\sqrt{3\mu_0/E} = 0.6$.

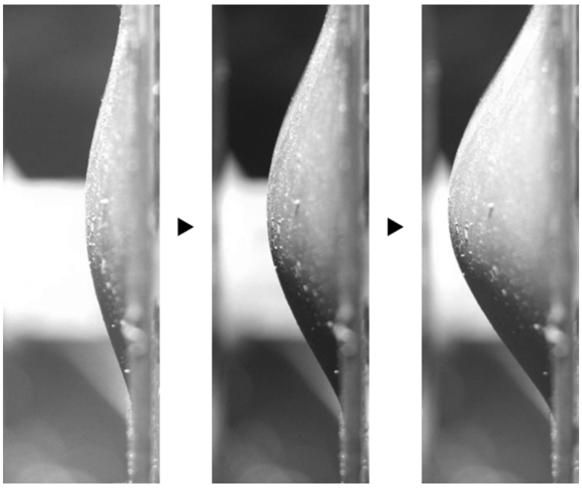

Figure 3. Domes formed by a membrane with $D/h = 10$; left to right: $H_0 = 50$, 80 and 110 kA m^{-1}.

membrane of a diameter D is cut from a SME sheet, then inserted in a matching hole in an organic glass plate of the same thickness and glued to the latter along the rim. The thus-prepared cell is installed in the middle of the gap (40 mm wide) of a laboratory home-made electromagnet, where the plate with the SME membrane is oriented parallel to the end walls (diameter 80 mm) of the pole pieces. As the gap is relatively wide, some non-uniformities of the field are conceivable. This implies that the samples of the smallest diameter ($D = 20$ mm) are the best with regard to the uniformity of the field. We found, however, that even for the largest samples ($D = 50$ mm), in which the bulk ponderomotive forces are undoubtedly present, they do not have a significant effect on the instability threshold. We ascribe this fact to their compensation due to the symmetry of the set-up.

Before each measurement the pole pieces are demagnetized in order that the departure field is always $H_0 = 0$; in the measurement the coil current is controlled with the accuracy 2% and is changed stepwise by 10 kA m^{-1} with respect to the field value. For a sample under test the up–down cycle of the field is run several times, the waiting time at each field value is 30 s, and it is 300 s in control runs. The sample is photographed from the direction across the gap with a digital camera that gives the accuracy of spatial measurements of about 0.1 mm. An example given in figure 3 illustrates the development of the dome when the field grows above the threshold.

The comparison with the theory is performed for the membrane of the smallest diameter available: $D = 20$ mm and $h = 1.9$ mm. Except for the geometry dimensions, our model requires just two material parameters of SME, both for the field-free state: the Young's modulus E and magnetic susceptibility χ. Their numerical values, $E = 30$ kPa and $\chi = 2.14$, determined in independent tests were substituted in our finite-element program. The results of this calculation are plotted in figure 4 against the properly scaled experimental ones. Note that the calculation does not incorporate any adjustable constants.

As seen, with respect to the critical field the difference between the calculated and measured values is about 15%,

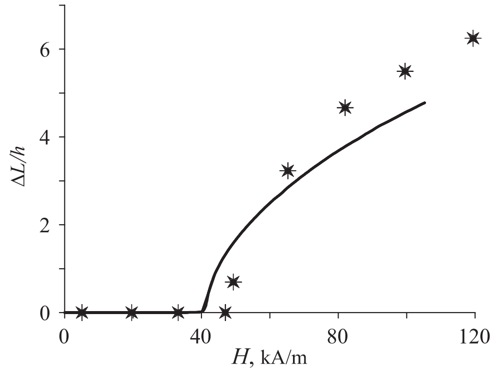

Figure 4. Theory versus experiment; the field dependence of the dome height for a membrane with $D = 20$ mm and $h = 1.9$ mm; measurement (points) and modelling (solid line); the material parameters of the sample are $\chi = 2.14$ and $E = 30$ kPa.

which is fairly good given the simplicity of the model. For the field dependence of the dome height ΔL the quantitative correspondence is poorer. We surmise two main causes for this. First is the non-uniformity of the field that induces bulk ponderomotive forces inside the SME sample. This effect is to a high extent compensated in the subcritical ($H_0 < H_{0c}$) regime where the sample is yet flat, but as soon as the membrane begins its left- or rightward bulging ($H_0 > H_{0c}$), the distribution of ponderomotive forces becomes unsymmetrical and seemingly affects the deformation. There is another source of discrepancies between theory and experiment which we associate with the field-induced anisotropy of the elastic modulus of the SME that we use; the presence of such anisotropy is already proven experimentally [17] but is not yet accounted for in our model.

In summary, a shape instability of a ferroelastic membrane fixed over its rim is predicted and proven to exist. In general features, the onset of this magnetomechanical effect follows the soft instability scenario (second-order transition) with respect to the field strength. With the aid of analytical estimations and numerical modelling, the main driving mechanism of the phenomenon is elucidated and a quantitative description of a field-induced SME membrane bulging is obtained. Some important issues, presently missing from the theory, are noted for further work.

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